Application of Multi-Factor Multi-Commodity Model for Cargo Dispatch Optimization

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In this article we describe the use of the multi-factor multi-commodity (MFMC) model in determining the optimal location for shipping a cargo, specifically the delivery of LNG. The MFMC model is able to capture the complex movements in forward prices that are difficult to capture using a single factor model, as well as correlations between different commodities. Such models can be used for a wide range of energy pricing and risk management tasks. These include spread option pricing, power plant valuations, and for the calculation of value-at-risk or earnings-at-risk type metrics for portfolios covering multiple commodities. The MFMC model is well suited to this type of analysis since it involves consideration of the commodity price at a number of locations, and we can consider each of these prices as describing separate but correlated commodities. For this example modeling the correlation is a key component since we would expect the price of the same commodity at different locations to be reasonably highly correlated. For this article we will simplify the problem to highlight the use of the MFMC model, and note that in reality a number of other factors dealing with the potential constraints and logistics of cargo transportation would also need to be considered.

Suppose a commodity can be shipped and sold to one of N different locations. The solving of the optimal location problem involves determining which location should be chosen in order to maximize the revenue for the holder of the flexibility. We refer to the price received for the commodity at a particular location as the *reference price*, which can be calculated as a linear combination of prices for that commodity at different locations, and may include other variables such as shipping costs. In the simplest case the reference price will be defined by a single commodity price. For example, if we deliver the LNG cargo to the US Gulf Coast the reference price may be simply defined as the NYMEX price for Henry Hub. However, if the cargo could be "split" at the destination and delivered to two nearby nodes with different spot prices the reference price may be a combination of the commodity prices at these different reference points. The reference price can be thought of as the net payoff (per unit volume) for the cargo at the destination. We define the optimum decision as the one which leads to the maximum payoff at the time of delivery of the cargo. One approach to determine the optimum location is to simulate the reference prices for the required locations using a MFMC model, and then determine which one has the greatest expected value.

Let

$F_j(t,T)$ be the forward price for commodity j ($j = 1,\ldots,M$) at time t and for maturity at time T; and

$S_j(T)$ be the spot price for commodity j at time T;

We define the reference price $m_i(T)$ for location i (i = 1,…,N) to be

$$m_i(T) = \sum_{j=1}^{M} w_{i,j} S_j(T) \qquad (1)$$

where $w_{i,j}$ represents the user defined weights of each commodity price.

The quantity $m_i(T)$ defines the price that would be received if the cargo was delivered to location i at time T and sold at the prevailing spot prices. Let h be the lead time between the decision date and the delivery date (T) at the destination. Of course, at time T - h when the decision is made we do not know what the spot prices in equation (1) will be, but our expectation of the spot price is given by $F_j(T - h, T)$. We therefore define the expected value of the reference price with lead time h as

$$m_i^*(h,T) = \sum_{j=1}^{M} w_{i,j} F_j(T\text{-}h,\ T) \qquad (2)$$

Therefore, in order to generate the reference price levels we must simulate the forward curve until time T-h to determine the forward price for maturity at time T. To simulate the forward curve, recall from our previous Masterclass that the forward curve dynamics for each commodity can be written in discretized form as the following;

$$\Delta \ln F_j(t, t+\tau_k) = -\tfrac{1}{2}\sum_{i=1}^{n} \sigma_{j,i}(t, t+\tau_k)^2 \Delta t + \sum_{i=1}^{n} \sigma_{j,i}(t, t+\tau_k)\Delta z_{j,i} \qquad (3)$$

where $\sigma_{j,i}(t, t+\tau_k)$ is the i^{th} (i = 1,…,n) volatility function for the j^{th} commodity, and $\Delta z_{j,i}$ is the corresponding random shock. For each time t that the simulation is performed we use equation (3) to generate the prices for a discrete set of relative maturities, τ_k, which typically represent the monthly points on the forward curve that cover the duration of the deal and where Δt is chosen such that the discretized equation is a sufficiently good approximation to the continuous time model. Typcially Δt would be one day for this type of application where gas prices are being considered. If we were modeling a more volatile commodity price, such as power, then the time step would generally be reduced to one hour.

In order to simulate the forward prices using this model the following inputs are required:

- an initial forward curve at time t for each commodity, that is, the set of prices $F_j(t,t+\tau_0), F_j(t,t+\tau_1), \ldots, F_j(t,t+\tau_N)$,
- the volatility functions $\sigma_{j,i}(t,t+\tau_k)$, and
- the correlation matrix between the different factors and different commodities.

The initial forward curve is generally obtainable as the latest available market quotes with the last two items typically estimated as described in our previous article. Note that the simulation involves generating multivariate normally distributed random numbers to capture the correlated Brownian increments, $\Delta z_{j,i}$. The standard procedure to do this is to first generate a set of univariate normally distributed random numbers using a standard algorithm such as the Box-Muller method[1]. Then we require the Cholesky decomposition, **L**, of the correlation matrix, **R**, that is

$$\mathbf{R} = \mathbf{L}\mathbf{L}^{\mathbf{T}} \qquad (4)$$

Having obtained **L** the set of appropriately correlated multivariate random numbers can be obtained by multiplying the univariate random numbers by the matrix **L**.

The basic algorithm to determine the optimum decision for shipping the cargo can therefore be summarized as follows:

1. Using equation (3), and starting from the current date, t, we simulate the monthly forward prices for each commodity until the decision date, i.e., $T - h$. Using these forward prices we evaluate the expected values of the reference prices, $m_i^*(h,T)$, via equation (2).
2. The optimum location is chosen as the one with the maximum reference price. We denote the index of this location as i_{MAX}, and the expected payoff is given by

$$m_{i_{MAX}}^*(h,T) = \max\left[m_1^*(h,T), \ldots, m_N^*(h,T)\right] \qquad (5)$$

[1] See section 4.11 of Clewlow and Strickland [1998] for a discussion of generating standard normal random numbers

3. The forward curve simulation continues as until the delivery date, *T*. The simulated forward price at time *T* for maturity *T* is simply the spot price, i.e., $F_j(T,T) = S_j(T)$, and so we can calculate the actual payoff which is $m_{i_{MAX}}(T) = m^*_{i_{MAX}}(0,T)$.

4. The above 3 steps are repeated for *M* simulations (e.g., *M* = 10,000), and for each simulation the delivery location, i_{MAX}, and the actual payoff at that location, $m_{i_{MAX}}(T)$, are stored for subsequent analysis.

The above steps outline a basic algorithm that can be used to solve the optimal dispatch problem. Depending on the commodity being modeled it may be necessary to incorporate other variables associated with delivery of the cargo into the calculation of the reference prices. One possible extension of the reference price calculation is to introduce a piecewise linear transformation. For the LNG shipping example used in this article we define a piecewise linear curve of up to five segments. In Figure 1, each segment of the piecewise linear curve is defined by three parameters: an intercept, a slope and an upper limit (i.e. I_i and S_i for *i* = 1,2,…,5; U_i = 1,…,4). The only exception to this is segment 5 where an upper limit does not exist (or it can be assumed the upper limit for this segment is infinity). To give an example of how the piecewise linear curve works, if the calculated reference price, m_i, falls within the range (U_{j-1}, U_j], the new value of the reference price, m_i', would be

$$m_i' = I_j + S_j \times m_i \tag{7}$$

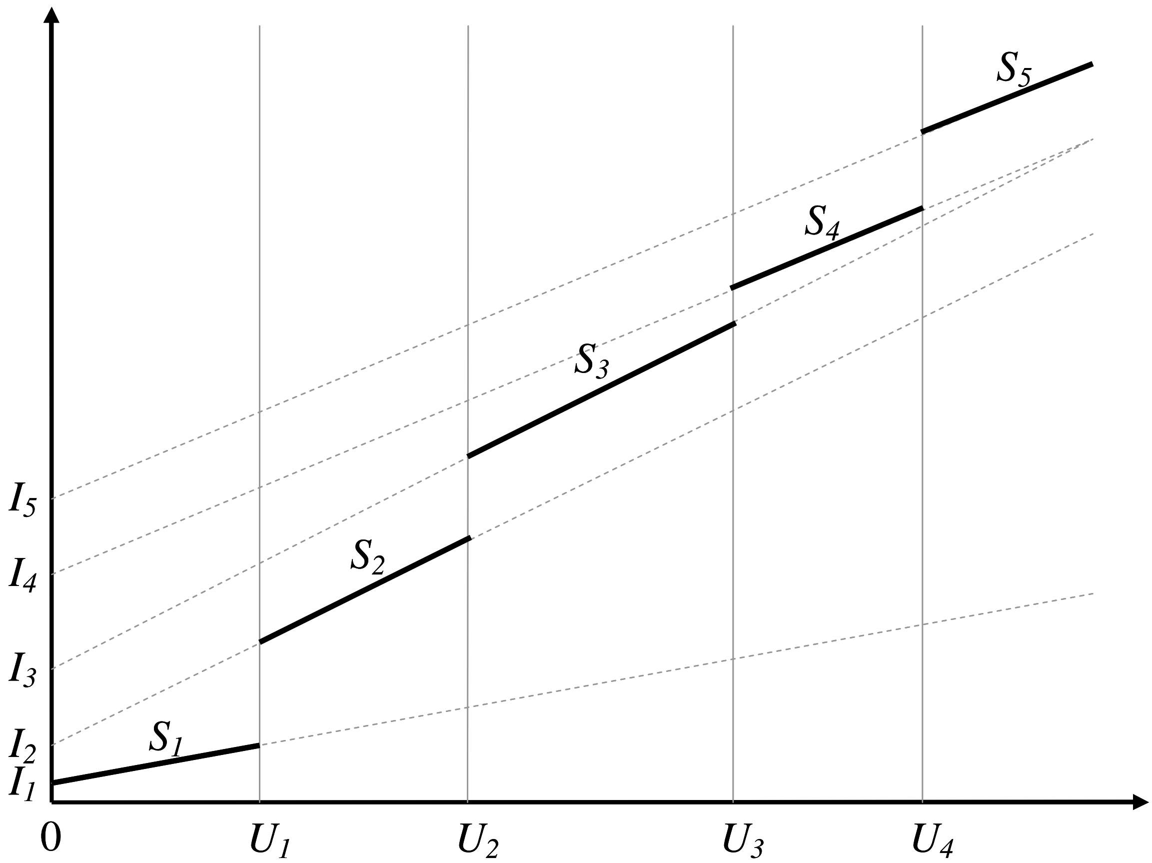

Figure 1 : An Illustration of a piecewise linear transformation of the reference price

In addition to the piecewise linear transformation, variables such as shipping costs and access costs per unit volume can be applied individually to the reference prices before they are compared in the decision calculation defined by equation (5).

In
Figure 2 we show a screenshot of how the inputs can be defined for the full reference price calculation as described above. For this example we have defined three locations, UK, USNE, and USGC. For each location a base commodity is specified, which define the unit and currency for that location. The reference price is then defined by entering the appropriate weights against the available commodities as defined in equation (1) or (2). In this example the user can choose from five possible commodity prices: NBP (UK National Balancing Point), HNG (Henry Hub), SOC (Southern California), TZ6 (TransCo Zone 6) and JCC (Japanese Crude Cocktail). For clarity we have assumed the price received in each region only depends on a single commodity price, either HNG or NBP. We have also defined a piecewise linear curve for the USGC location in this example to illustrate the inputs but also to ensure the reference price calculations for the USNE and USGC locations are different (since they both depend on HNG prices). Finally any additional Shipping or Access Point charges can be subtracted.

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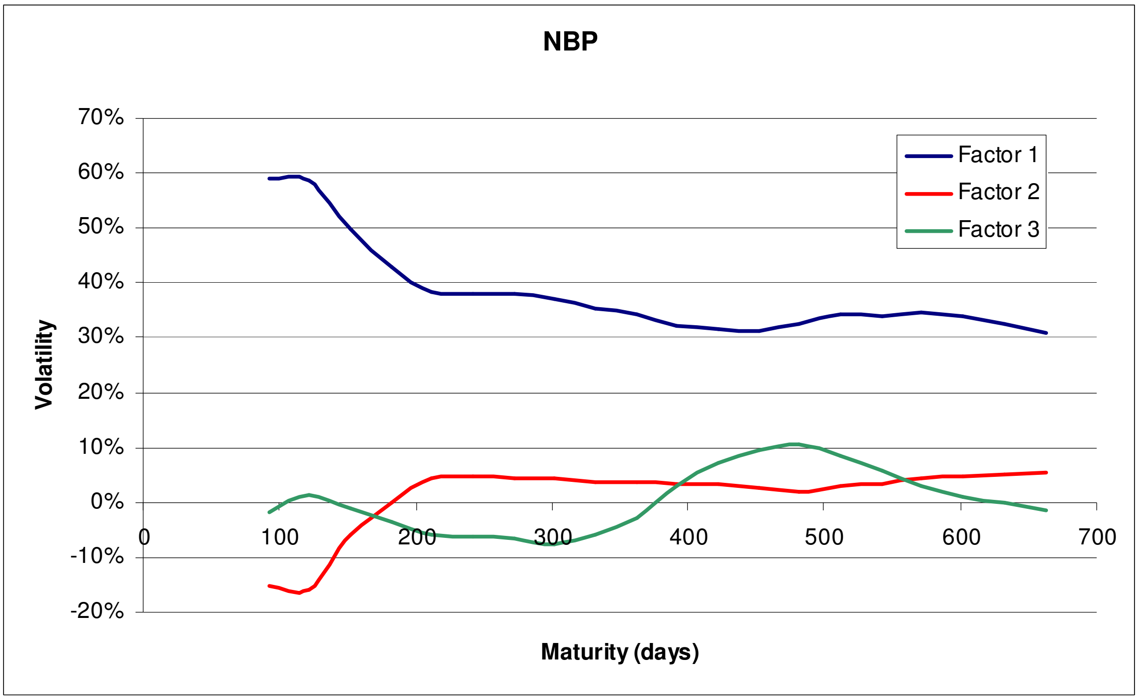
Marker Specifications

	UK	USNE	USWC	USGC	Asia	Japan
Include	TRUE	TRUE	FALSE	TRUE	FALSE	FALSE
Base Commodity	NBP	HNG	SOC	HNG	JCC	JCC
Base Unit	Therm	MMBtu		MMBtu		
Base Currency	pence	USD		USD		
NBP	0.85	0.00	0.00	0.00	0.00	0.00
HNG	0.00	0.98	0.00	0.81	0.00	0.00
SOC	0.00	0.00	0.00	0.00	0.00	0.00
TZ6	0.00	0.00	0.00	0.00	0.00	0.00
JCC	0.00	0.00	0.00	0.00	0.00	0.00
Variable Charge	0.00	0.00	0.00	0.00	0.00	0.00
Use Linear Transformation	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE
Intercept 1	0	0	0	6	0	0
Slope 1	0	0	0	0	0	0
Upper Limit 1	0	0	0	7	0	0
Intercept 2	0	0	0	0	0	0
Slope 2	0	0	0	1.5	0	0
Upper Limit 2	0	0	0	8.5	0	0
Intercept 3	0	0	0	1	0	0
Slope 3	0	0	0	1.25	0	0
Upper Limit 3	0	0	0	9	0	0
Intercept 4	0	0	0	1.5	0	0
Slope 4	0	0	0	1	0	0
Upper Limit 4	0	0	0	10.5	0	0
Intercept 5	0	0	0	14	0	0
Slope 5	0	0	0	0	0	0
Per Unit Cost (in USD/MMBtu)						
Shipping	1.90	1.20	0.00	1.50	0.59	0.00
Access Point	0.00	0.00	0.00	0.00	0.00	0.00

NUM

Figure 2 : Inputs for reference price specification

Consistent with the seasonality discussion in our last article, for the forward curve simulation we can define a set of seasonal volatility factors for the NBP and HNG prices. In Figure 3 we illustrate the first three factors for each commodity for the first quarter (results for the other quarters are similar).



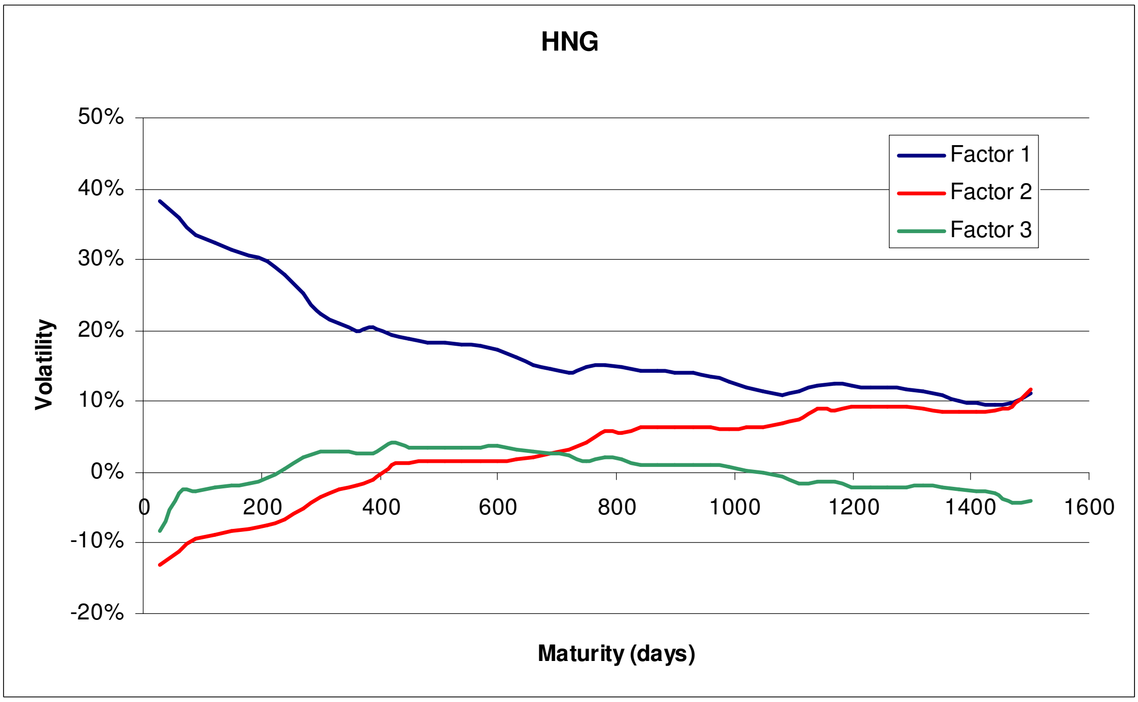


Figure 3: The first three volatility factors for each commodity for Q1

The simulation of the forward prices also requires an initial forward curve. For this example we consider a range of decision and delivery dates covering 2008 and 2009, so the forward curves must cover this period also. The forward curves used are illustrated in Figure 4. Note that for ease of comparison we have converted the NBP prices from p/Them to USD/MMBtu using a currency conversion rate of 2 USD/GBP.

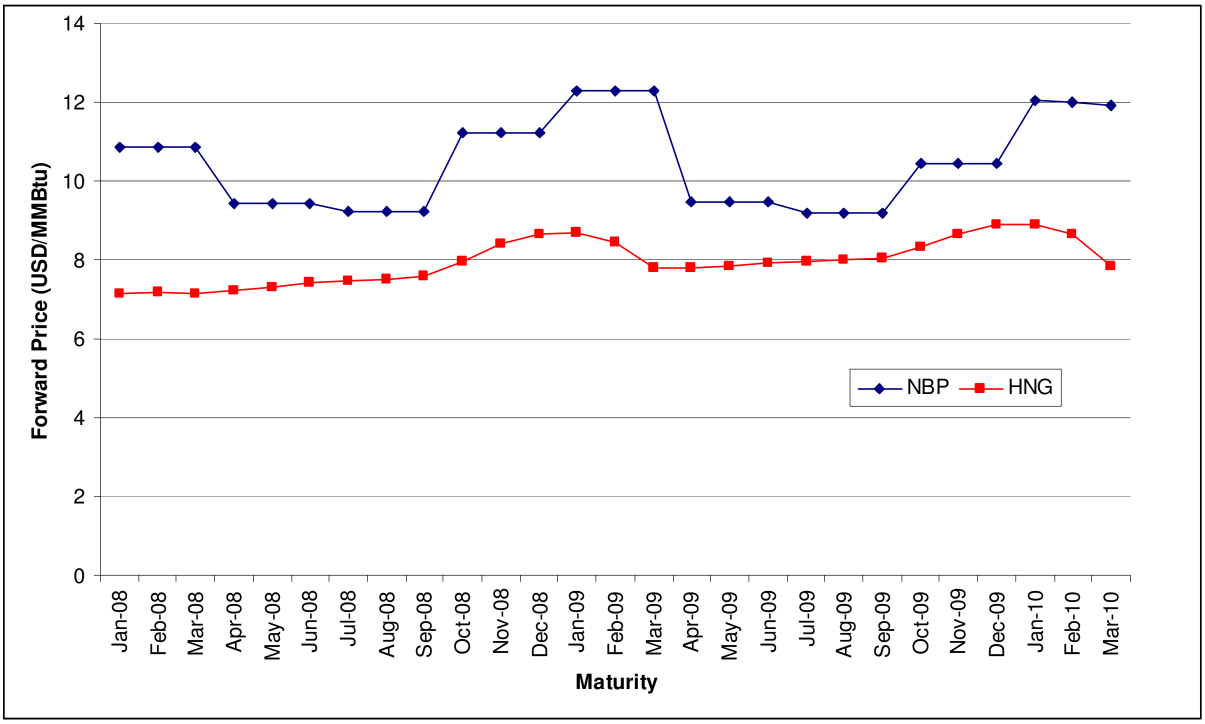


Figure 4 : Initial forward curves used for the simulation.

There are a number of possible outputs that can be obtained from the forward curve simulation and subsequent analysis. The key output is the probability of each location being the optimal location for the specified decision date and lead time. In Figure 5 we illustrate the probabilities for a range of decision dates from Jan-2008 to Dec-2009, and for two different lead times. This example uses 10,000 simulations.

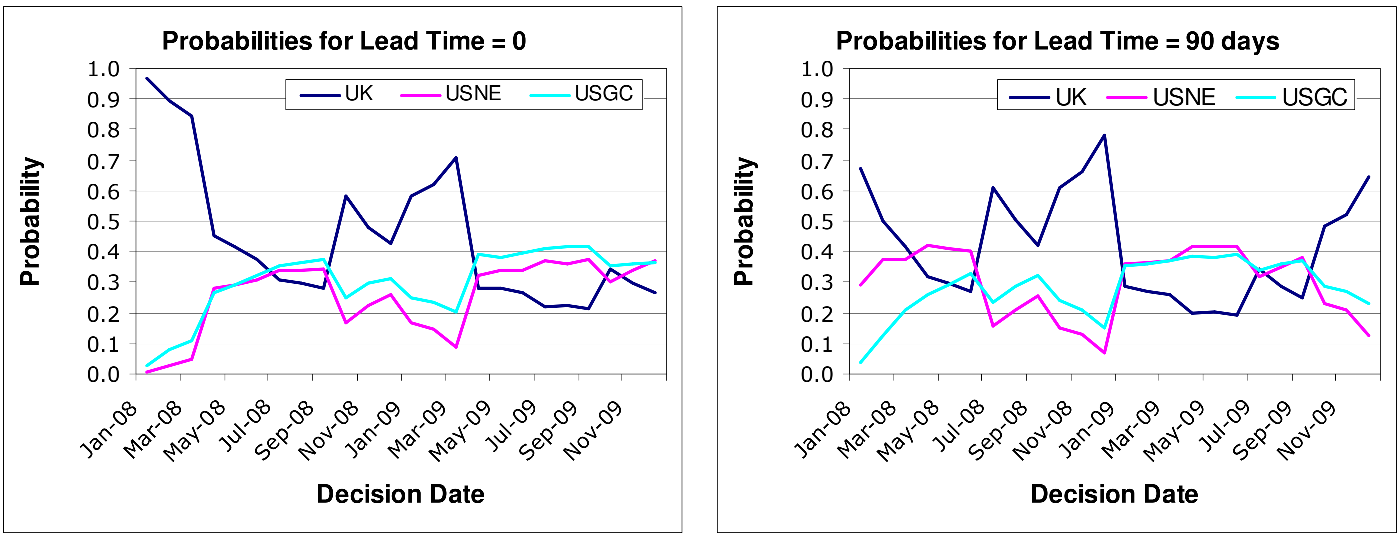


Figure 5 : Output probabilities for each location and user defined lead time

The lead time of zero implies that the decision date and delivery date are identical, so the first panel in figure 5 illustrates the results if the decision were made on the simulated spot price at each location. While this assumption is not realistic for actual cargoes shipped (at least for our example of an LNG cargo) it is useful to calculate this result as it shows what the decision probabilities would be with perfect foresight. The second panel in figure 5 plots the results for a lead time of 90 days. Note that the x-axis is the decision date, so when comparing the two panels we need to keep this in mind – for example, in the right hand panel (i.e., using a 90 day lead time) the peak in the probability for the UK destination with a decision date of Dec 2008 corresponds to a decision date of Mar 2009 in the left hand plot with zero lead time. Apart from this horizontal shift it can be seen that the results for the two cases are similar in general shape, yet show significant differences at some points. To interpret these results consider the right hand plot. The lead time of 90 days is the time between when a decision is made on the destination for the cargo and the arrival at the destination. So again, if we consider the points at Dec 2008, the plot shows the probability of each location having the maximum payoff when the cargo is delivered to the destination in Mar 2009. In this case the UK has a very high probability of having the highest payoff, so a cargo that could be shipped at this time should be sent to the UK as opposed to one of the US locations. At other times of the year there is little difference between the locations, or the relationship is reversed.

Clearly, as well as the probability of a location having the highest payoff, another important output from the model is the expected payoff (per unit volume), that is, the average value of the simulated payoffs, $m_{i_{MAX}}(T)$ defined earlier. We would expect that if the lead time is zero then the payoffs will be a maximum since the decision for the optimal delivery location is based on the spot price itself. For any lead time greater than zero the payoffs will decrease since the decision is based on a simulated forward price which will generally be different from the eventual spot price. In other words, in the case of zero lead time the decisions will be perfect (since they are based on the actual payoff), but the further away the decision date is from the actual payoff the more imperfect the decisions will become. This is illustrated clearly in Figure 6.

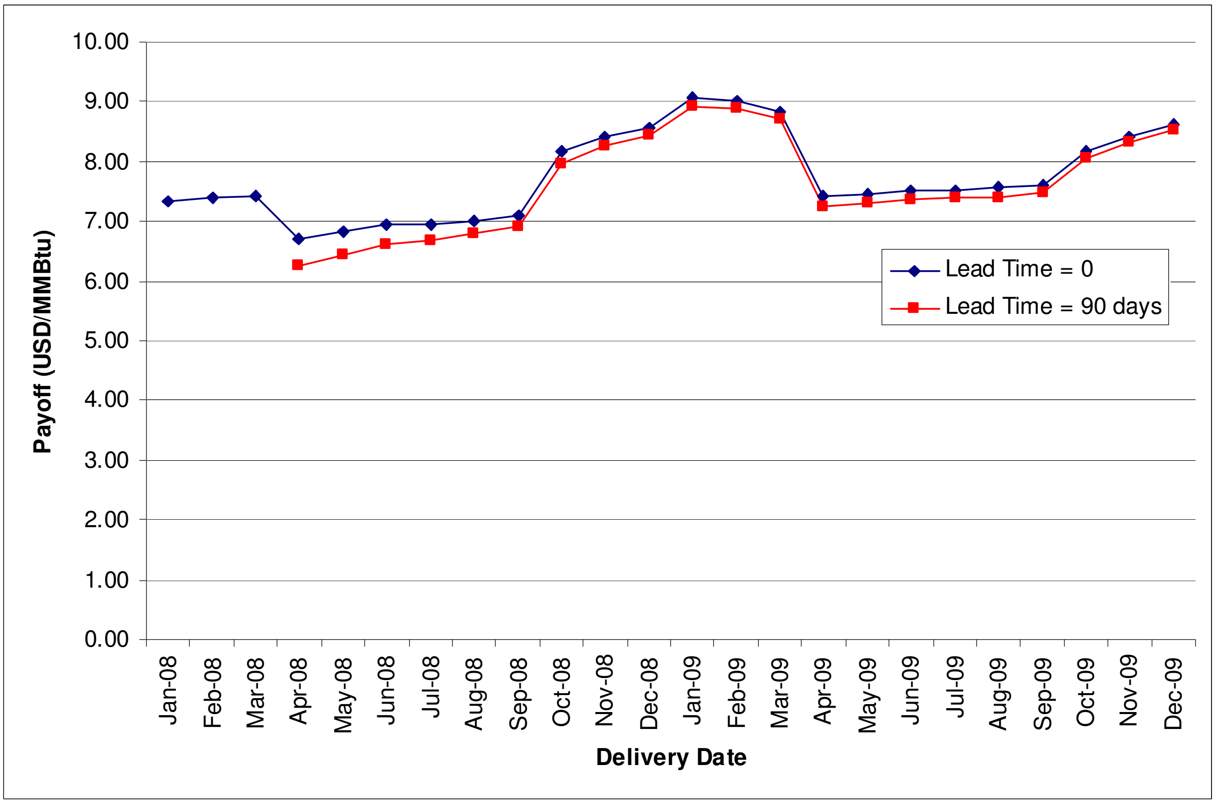


Figure 6 : Expected payoffs for each delivery date. Note that for Lead Time = 90 the first delivery is in Apr 2008, since the decision dates used for this example start in Jan 2008.

The results in Figure 6 show the expected payoff across all locations. From a more detailed analysis of the simulations we could calculate the conditional expected payoffs, that is, the expected payoff for each location conditioned on the cargo being shipped to that location. Additional outputs such as distributions of conditional or unconditional payoffs can also be derived from the simulation results.

As a final point we note that the analysis described above involves calculating payoffs effectively based on the spot price. In reality the seller would often hedge some or all of their commodity shipment in the forward market. This adds another level of complexity to the model, but simulation using the MFMC framework is still an ideal approach to model the underlying prices. In particular a simulation based approach is ideal for modeling the distributions of results, which provides a straightforward mechanism for comparing the effectiveness of different hedging strategies.

References

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Lacima Group is a specialist provider of energy and commodity pricing, valuation and risk management software and advisory services. Based on its internationally acclaimed research in energy risk modeling, Lacima offers integrated risk management applications to address valuation, market and credit risk or the flexibility of stand-alone solutions for swing, storage and generation assets. These solutions help energy producers, retailers, distributors, large consumers and financial institutions to value and manage risk associated with complex derivative contracts and physical assets across multiple commodities and regions in a cost-effective manner.

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