Measuring & Capturing Value in the Flexibility of Gas Swing Contracts

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Gas swing contracts are well established, but it is only recently that techniques have been developed to measure and capture the value in their flexibility. John Breslin, Les Clewlow, Chris Strickland and Daniel van der Zee of Lacima Group discuss these techniques.

Gas swing contracts for the purchase and sale of gas embed flexibility as to the volume of gas traded and have been present in the natural gas markets for many years. However, it is only recently that techniques have been developed that can capture the value in the flexibility of these contracts and permit realistic incorporation into at-risk calculations.

A typical swing contract is an agreement between a supplier and a purchaser for the delivery of variable daily quantities of energy (typically gas or power), between specified minimum and maximum daily limits, over a certain number of years at a specified set of contract prices. While swing contracts have been used for many years to manage the inherent uncertainty of gas supply and demand, it is only in recent years with deregulation of the energy markets that there has been an interest in understanding and valuing the optionality contained in these contracts. In this article we outline the key features of typical swing contracts, describe how the value in these contracts can be determined, and also how the constraints on the contract impact the optimal exercise strategy.

The main features of swing contracts which make them difficult to value and risk manage are the constraints on the quantity of gas which can be taken. The main constraint is that in each gas year there is a minimum volume of gas (termed take-or-pay or minimum bill) for which the buyer will be charged, at the end of the year (or "penalty date"), regardless of the actual quantity of gas taken. Typically, there is also a maximum annual quantity which can be taken. The minimum bill or take-or-pay level is usually defined as a percentage of the notional annual quantity which is called the annual contract quantity. So to define a simple swing contract the following inputs are generally required:

- Start date and end date
- Penalty dates (usually end of the gas years but can be monthly or quarterly)
- Annual/period contract quantity (ACQ/PCQ)
- Minimum bill (MB)
- Annual/period maximum quantity
- Daily contract quantity (DCQ)
- Daily minimum and daily maximum
- Contract price (usually constant or deterministic, but can be linked to the prices of other variables)

Typically the MB and annual maximum quantity will be defined as a percentage of the ACQ and the daily minimum and maximum as a percentage of DCQ.

Swing contracts can be viewed as a mixture of a daily swap and a strip of daily call options. In the absence of a take-or-pay constraint, that is, when MB = 0% and no annual maximum constraint, the optimal strategy each day is to purchase the maximum allowable quantity of gas when the market price is above the contract price and nothing otherwise. In this case the contract has the maximum amount of flexibility and value and is equivalent to a strip of European call options. If the take-or-pay level is equal to the maximum amount that can be taken (minimum bill = maximum annual quantity = sum of daily maxima) the optimal strategy each day is to purchase the maximum allowable quantity of gas regardless of the market price. In this case the contract has the minimum amount of flexibility and value and is equivalent to a swap. If the take-or-pay level is less than the maximum amount that can be taken and greater than zero (for example, MB = 80% of maximum annual quantity) then the optimal strategy is much more complicated than that for either a strip of call options or a swap. Each daily decision (to take or not to take gas) creates an immediate positive or negative cash flow as well as a change in the value of the remainder of the contract; taking gas today decreases the number of future forced takes to avoid the MB penalty, but also decreases the future allowed takes before the period maximum is reached.

Given a market price (forward curve), some assumptions about the process followed by the underlying spot price, and the contract parameters, it is possible to determine the optimal decision and associated value for the contract at any point in time. A reasonably realistic model for gas spot prices is the mean reverting spot price model first described by Schwartz [1997] and later extended by Clewlow and Strickland [1999, 2000] to be consistent with the market forward curve and spot volatilities. This model can be described by the following equation:

$$\frac{dS(t)}{S(t)} = \left[\frac{\partial \ln F(0,t)}{\partial t} + \alpha \left[\ln F(0,t) - \ln S(t) \right] + \frac{1}{2} \alpha \int_0^t \sigma(u)^2 \exp\left(-2\alpha(t-u) \right) du \right] dt + \sigma(t) dz(t)$$

Where $S(t)$ is the spot price at time t, $F(t,T)$ is the forward price at time t with maturity date T, α is the mean reversion rate, $\sigma(t)$ is the spot price volatility at time t, and $dz(t)$ is an increment in a Wiener process. Clewlow and Strickland [1999, 2000] also describe how a trinomial tree can be built to represent this model[1] and a general method for solving path dependent derivatives such as swing contracts based on the trinomial tree. This method allows the optimal decision for the swing contract to be obtained. The optimal decision is defined as that which leads to the maximum expected value. For the simple take-or-pay contract, the decision and value will depend on the current spot price and the volume of gas already taken, so we can define the decision and value surfaces as functions of these variables. In Figures 1 and 2 we illustrate some sample surfaces for a simple 1-year contract with DCQ = daily maximum = 1, daily minimum = 0, ACQ = annual maximum = 365, and MB = 80% (equivalent to 292 units of volume), and for which the contract price and forward price are both flat and equal to 100 for the entire year.

The surfaces in Figure 1 illustrate the key features of a typical decision surface. We denote the critical price as the spot price above which it is optimal to take the maximum daily quantity. In the upper left "Day 0" (pricing date) plot, note that at low "period-to-date"[2] values the critical spot price may be less than the contract strike price, even though this results in an immediate loss. This non-intuitive result is due to the expectation that future spot prices can be even lower and cause greater forced losses in order to meet the MB. So it is better to take a small loss now to reduce to probability of having to take a larger loss later. Below the critical price the expectation is that the loss incurred by taking gas today is greater than the expected future loss from taking when the spot price is higher. This is affected by the mean reversion rate – if the mean reversion rate is high then the expected number of lower spot prices decreases, if the reversion rate is low then the expected number of lower spot prices increases.

Notice that the "Day 0" surface also gives the optimal decisions for "period-to-date" values greater than zero. This would apply if the buyer and seller of the contract were to agree to an effective existing volume of gas having been taken. As the "period-to-date" increases the critical price also increases. This is because the maximum annual take is 365 so the holder of the contract will only be able to take gas on at most (365 – "period-to-date") days. This is reflected in the higher critical price which indicates that the holder must be more selective about which days to take gas. In effect having fewer days on which they can take gas means there will be more days they can choose not to take gas, which will obviously be those days with low relative prices.

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[1] The description assumes a constant spot volatility but the method is straightforward to generalise to a spot volatility which is a general function of time.

[2] The period-to-date is the volume of gas taken in the current penalty period to the current date.

For a non-zero MB condition there will be a point in time at which the decisions become constrained and the optimal decision is to take gas irrespective of the spot price. For this example the MB condition of 80% first shows at Day 73 (20% into the year), and the effect is clear at Day 219 (60% into the year) where the critical price is at zero for all "period-to-date" levels below 40% of the maximum annual take (146). This level of "period-to-date" leaves 40% of the maximum annual take to make to reach MB in the 40% of the year remaining.

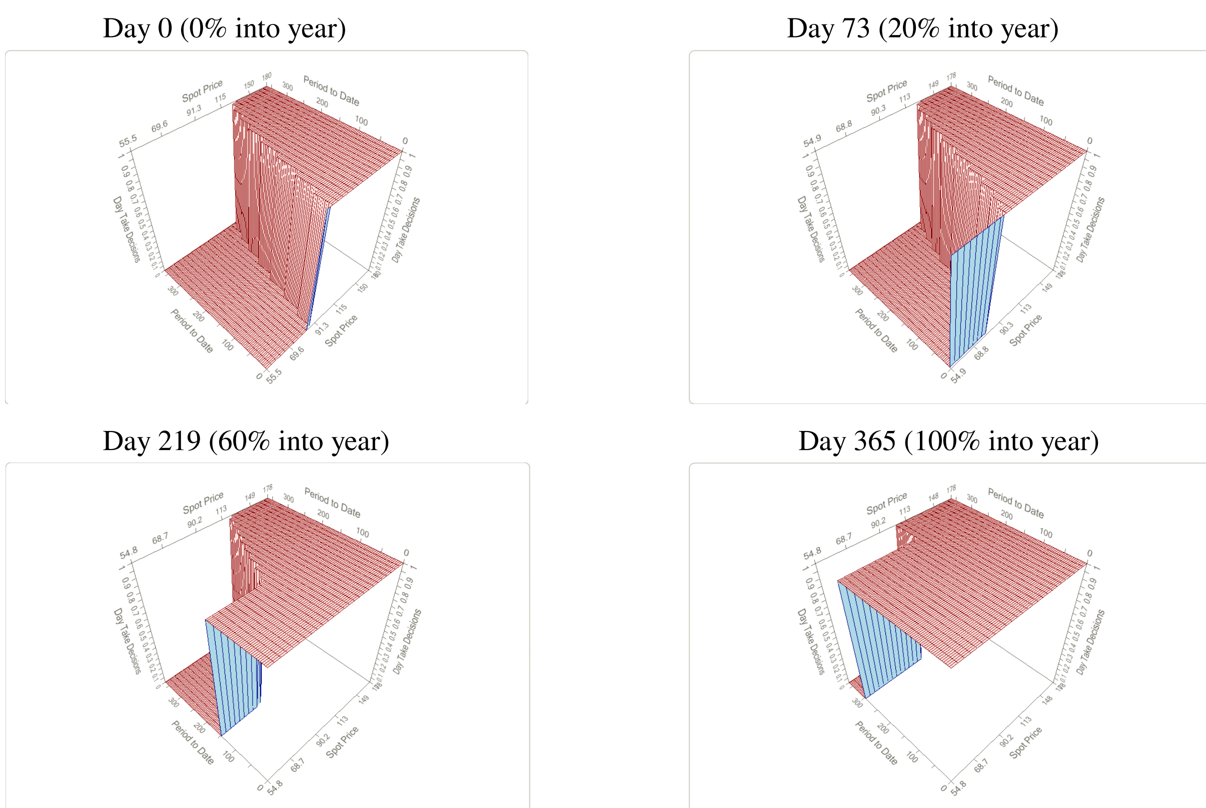

Figure 1: Example Decision Surfaces

In Figure 2 we show the corresponding value surfaces for the same 4 days as in figure 1. The first plot at Day 0 illustrates a typical unconstrained value surface, where highest value is found for high spot price and low "period-to-date" values. This is where the expectation is highest for positive future cashflows – spot prices much greater than the contract price. Conversely, the lowest value occurs for the lowest spot price and lowest "period-to-date" – this is where the expectation is highest for negative future cashflows.

As we move forward in time and the system becomes constrained by the Minimum Bill condition (beyond day 73) the impact on the value surface becomes clear – values in the constrained region start to decrease. In these cases, as the "period-to-date" volume decreases the values become more negative. This arises because of the MB constraint which means the buyer is forced to pay for the gas even if it is not taken. This can be viewed as a penalty that is imposed when the total gas take for the contract is less than the MB level. The effect of this is clearly evident at day 219. On the final time step, if the "period-to-date" value is greater than MB the value is zero, but if it is less than MB then the penalty is proportional to the difference between the MB and the volume of gas taken.

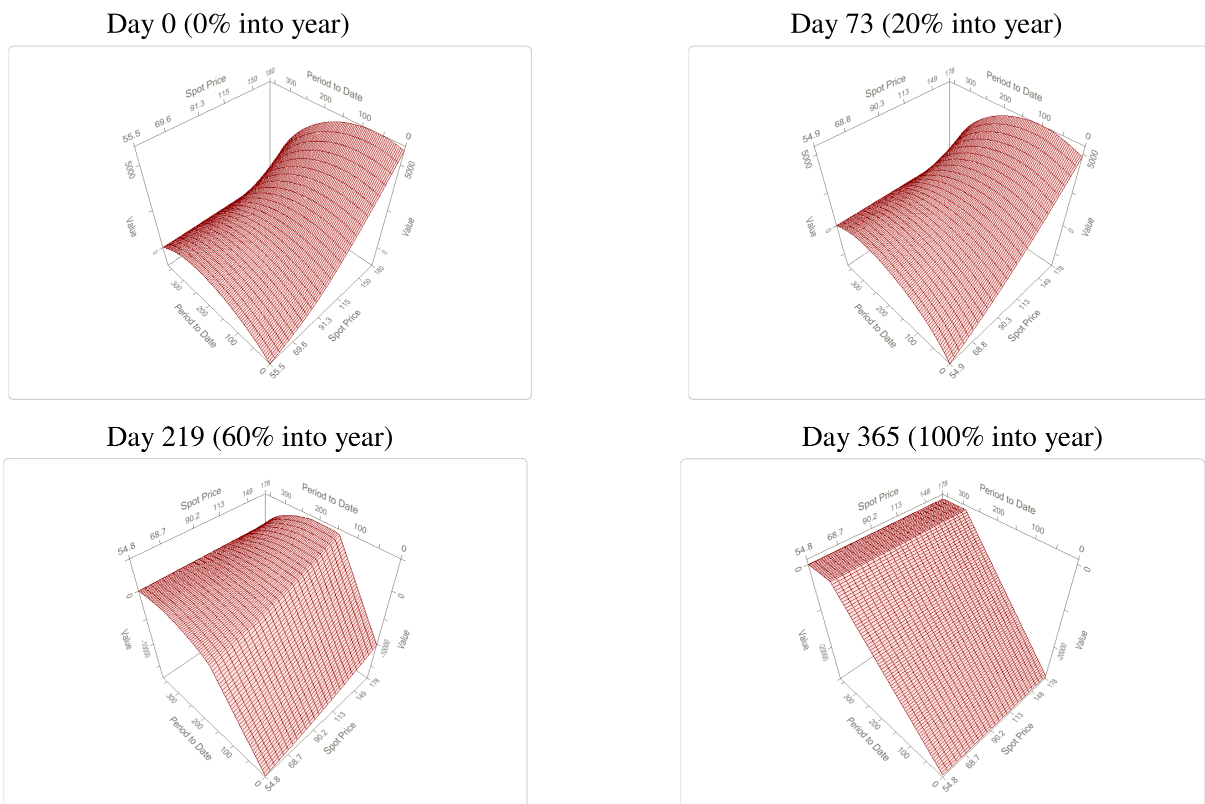

Figure 2: Example Value Surfaces

In addition to the basic contract parameters described above other common features for swing contracts are make-up and carry-forward. These are used for multi-year contracts and add another level of complexity to the analysis. In years where the gas taken is less than minimum bill the shortfall is added to a make-up bank. In later years where the gas taken is greater than some reference level (typically minimum bill or ACQ) the additional gas can be taken free of charge from the make-up bank. In essence this free gas has already been paid for when the MB penalty was applied in the year that the shortfall occurred. Similarly in years where the gas taken is greater than some reference level (carry-forward base) which is typically ACQ, the excess gas is added to the carry-forward bank. In later years carry-forward bank gas can be used to reduce the minimum bill for that year. The amount that can be used from the make-up and carry-forward banks each year is usually limited by a recovery limit which is typically around 20% of ACQ.

Depending on the relative levels of the contract price and forward price in each year, it can make sense to take less than MB in an earlier year to increase the volume in the make-up bank, and then to use this "free" gas at a later date. While it is possible account for make-up and carry-forward to determine an "intrinsic" exercise strategy for a given a forward price, the full value of the contract becomes very difficult to calculate without the use of a full optimization model.

Figure 3 shows an example of a simulated optimal strategy with make-up and carry-forward. The example is a six year contract with the following features:

- Daily maximum = 1
- Daily min = 0
- Annual maximum = ACQ = 365
- Minimum bill = 75% = 273.75
- Contract price = 100
- Make-up recovery limit = 20% ACQ = 73
- Carry-forward base = 80% ACQ = 292
- Carry-forward recovery limit = 20% ACQ = 73

The forward curve used in the example is shown in Figure 4.

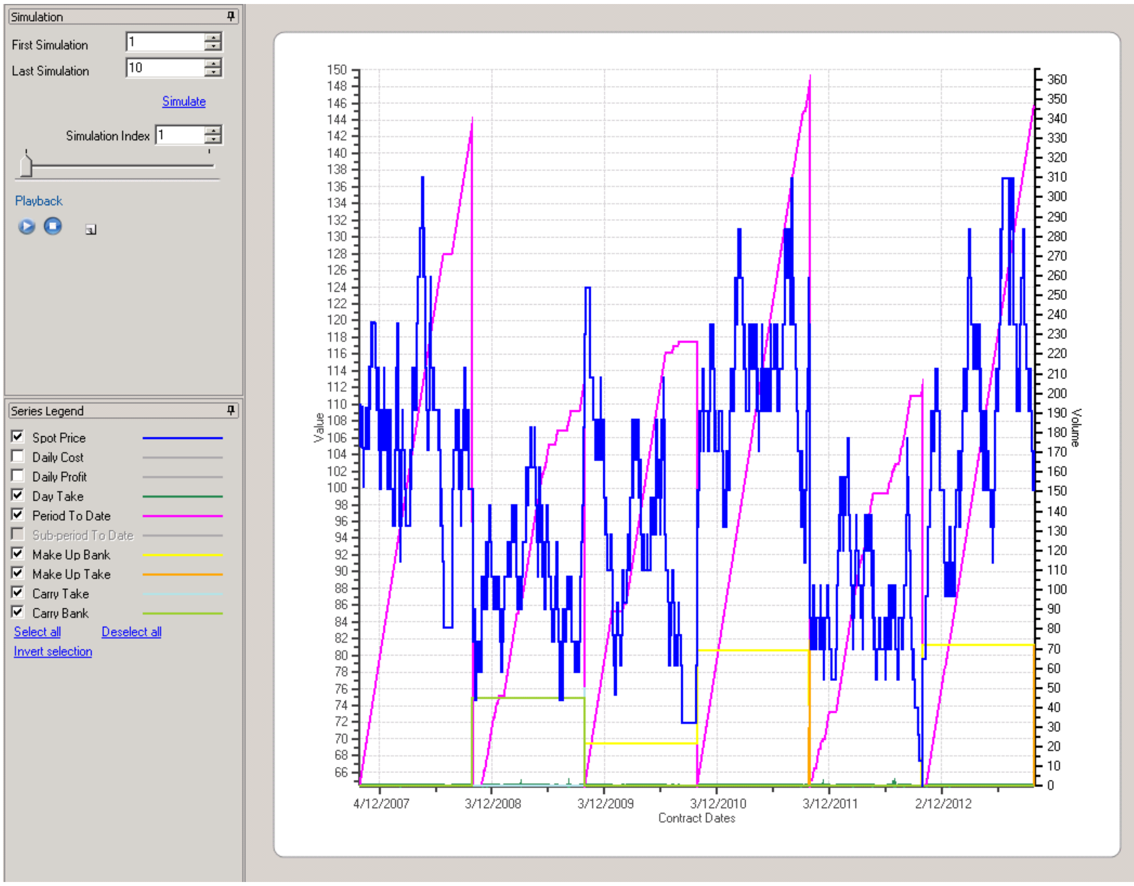

Figure 3: Simulated Optimal Strategy with Make-Up and Carry Forward

In the first year of the example (October 1, 2007 to September 30, 2008) the forward curve is above the contract price (we refer to this as in-the-money). If the spot price followed the forward curve (we call this the intrinsic strategy) – a single simulation of the spot price is represented by the blue line in Figure 3 – then the optimal strategy would be to take the maximum possible (365) and create 73 units of carry forward (365-292). However, the simulated spot price in Figure 3 is sometime below the contract price and so the simulated take is 337 which creates 45 units of carry forward.

In the second year the forward curve is below the contract price (out-of-the-money) and so the optimal intrinsic strategy would be to use the carry forward bank to reduce the minimum bill to 273.75 – 73 = 200.75 and then take 127.75 to create make up bank of 73 (the maximum that can be recovered). The simulated spot price is sometimes above the strike price so the simulated take is 207 creating make up bank of 21.75 (273.75 – 45 – 207).

The third year is also out-of-the-money so the intrinsic strategy is to take minimum bill plus the amount of gas in the make up bank which will be free giving a take of 273.75 + 73 =

346.75. With the simulated spot price spending a significant part of the year above the contract price the actual strategy is to take 226 and increase the make up bank to 69.5 (21.75 + (273.75 – 226)).

The fourth year is in-the-money so the optimal intrinsic strategy is to take the maximum quantity of gas (365) and create another 73 units of carry forward. In the simulation, the spot price is below the contract price for a few days so the take is only 357 and the full 69.5 units of make up bank are taken as free gas.

The fifth year is out-of-money so the intrinsic strategy would be the same as in year two. In the simulation, some days are actually in-the-money so the take is 202 which creates 71.75 of make up.

The final year is in-the-money so the intrinsic strategy would be to take the maximum possible with 73 units being taken free from the make up bank. The simulated price is below the contract price for a significant part of the year so the simulated optimal strategy is to take 346.75 of which the 71.75 in the make up bank is taken for free.

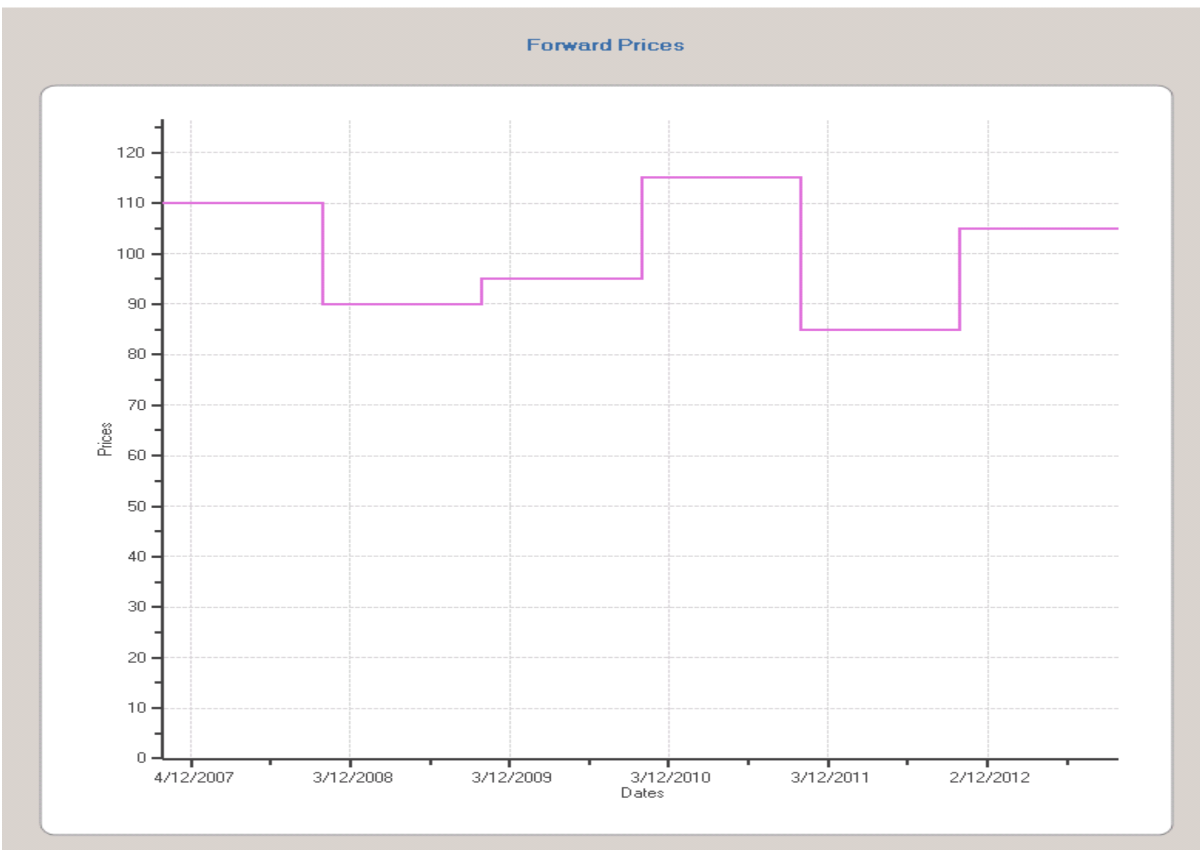

Figure 4: Forward Curve for Make-Up and Carry Forward Example

This simple example shows that even the intrinsic strategy, which assumes that the future spot price is known, is complex to solve. However, the intrinsic strategy unlocks only part of the value in a swing contract. The full value can only be realized when taking account of the uncertainty in spot prices, and can only be determined using an efficient and sophisticated numerical solver to evaluate all possible decisions.

Les Clewlow and Chris Strickland are the founders and Directors of Lacima Group, where John Breslin and Daniel van der Zee are Principals

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About Lacima Group

Lacima Group is a specialist provider of energy and commodity pricing, valuation and risk management software and advisory services. Based on its internationally acclaimed research in energy risk modelling, Lacima offers integrated risk management applications to address valuation, market and credit risk or the flexibility of stand-alone solutions for swing, storage and generation assets. These solutions help energy producers, retailers, distributors, large consumers and financial institutions to value and manage risk associated with complex derivative contracts and physical assets across multiple commodities and regions in a cost-effective manner.

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